MATH 601: Quiz 2 (9/10/2012)

Name: UIN:

Problem 1: Find a parametric representation (using a parameter t) of a line that goes through points $P = (1, 1)$ and $Q = (3, 1)$.

Answer: The located vector pointing from P to Q is given by $\overrightarrow{PQ} = Q - P = (2, 0)$. Then, according to what we've discussed in class, one parameterization of the line that goes through both of these points is

$$x(t) = P + t\,\overrightarrow{PQ} = (1, 1) + t(2, 0).$$

Note, in particular, that $x(0) = P$ and $x(1) = Q$ – in other words, for $t = 0$ and $t = 1$ the line is at P and Q, respectively. Of course, since the parameterization above describes a line, and we have just shown that the line goes through both P and Q, we have just found the single, unique line that goes through these two points.

Problem 2: In class we have discussed that the collection of points $\{x\}$ that make up a hyperplane all satisfy the equation $a \cdot x = \alpha$ for some $a \in \mathbb{R}^n, \alpha \in \mathbb{R}$. Providing a, α then completely describes a hyperplane. Find $a \in \mathbb{R}^3, \alpha \in \mathbb{R}$ that describes the hyperplane in $\mathbb{R}^3$ (i.e., a plane) that contains the point $P = (1, 1, 1)$ and that is perpendicular to the vector $u = (2, 2, 2)$.

Answer: In class we have shown that if $H = \{x\}$ is the hyperplane characterized by a, α, then a is a vector that is in fact perpendicular to the plane because it is perpendicular to the vector $\overrightarrow{PQ}$ for any arbitrarily chosen two points $P, Q \in H$.
Here, we are asked to find a plane that is perpendicular to the vector $u = (2, 2, 2)$, i.e., u is perpendicular to the plane. So we can take $a = u$ and we already know that the points x of the plane have to satisfy

$$a \cdot x = 2x_1 + 2x_2 + 2x_3 = \alpha$$

for an as yet unknown value of α. On the other hand, we know that the plane must go through the point $P = (1, 1, 1)$. This means that P is a point of this plane, so we know that

$$a \cdot P = (2, 2, 2) \cdot (1, 1, 1) = 6 = \alpha$$

as well. This provides us with the value for α.
To sum up, one characterization of the plane is given by $a = (1, 1, 1), \alpha = 6$.
Note that there are many other parameterizations. For example, we could have chosen $a = (3, 3, 3), \alpha = 18$, or $a = (-1, -1, -1), \alpha = -6$; both of these choices characterize the very same plane as the one we found above, just in a different way.
