Optimizing the Placement of Oil Wells

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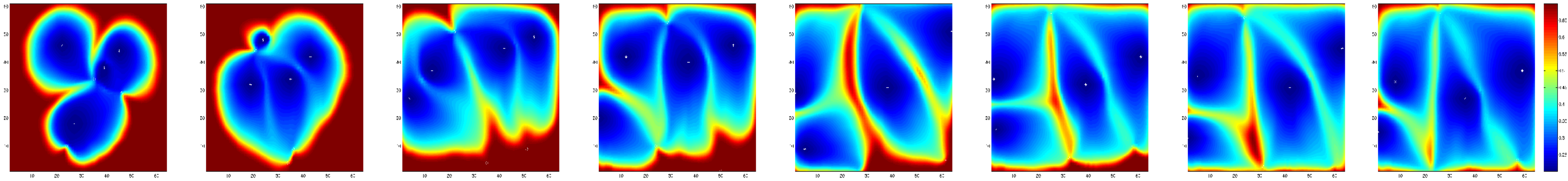

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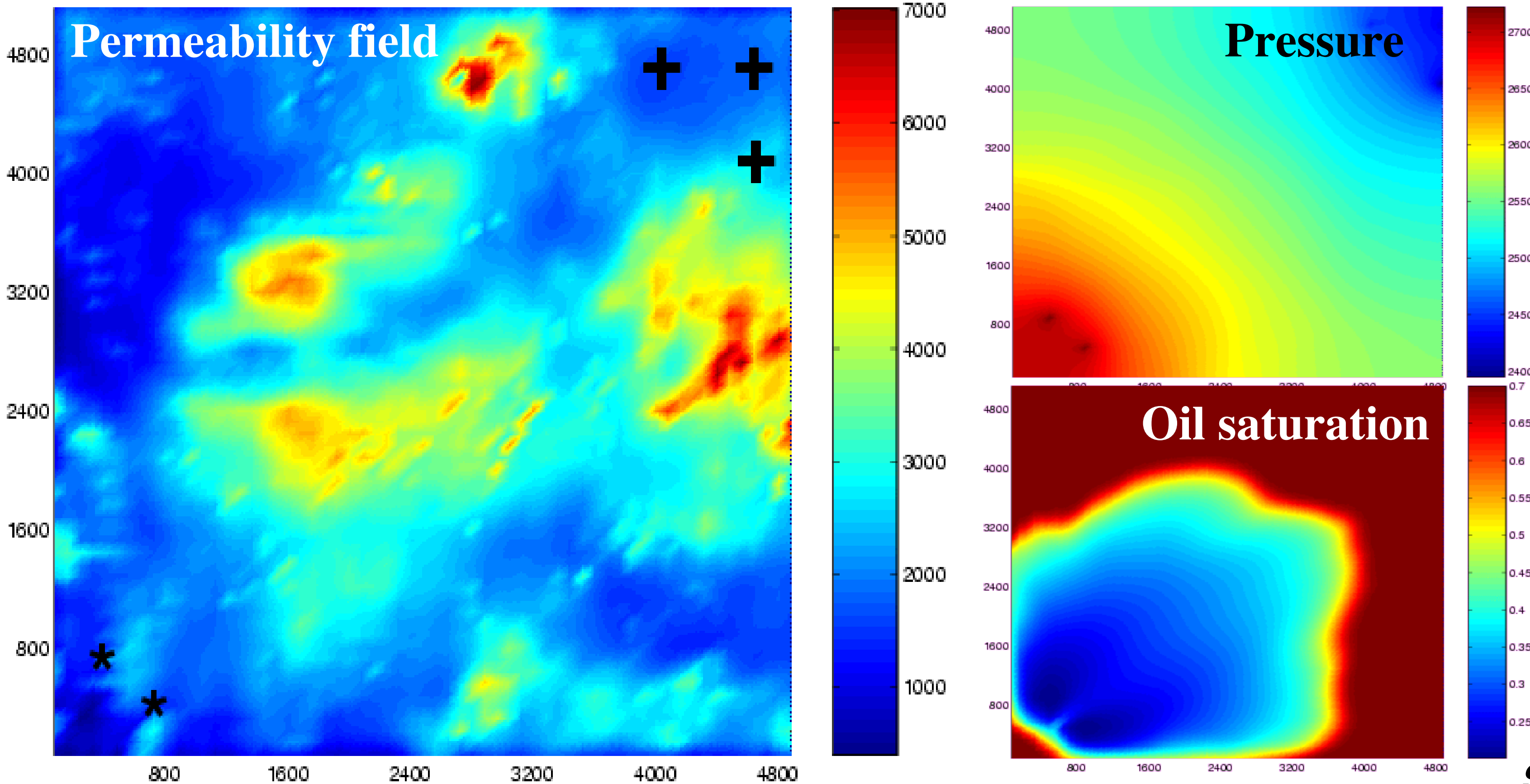
Simplified Setting

Given: An (unproduced) oil field; permeability and other material properties; locations of a few producer/injector wells

Question: Where is the best place for a third injector?

Goal: To have fully automatic methods of optimization!



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Mathematical description

Describe production from an oil field using the following components:

- Multiphase (oil/water) model
- Well model for given well locations

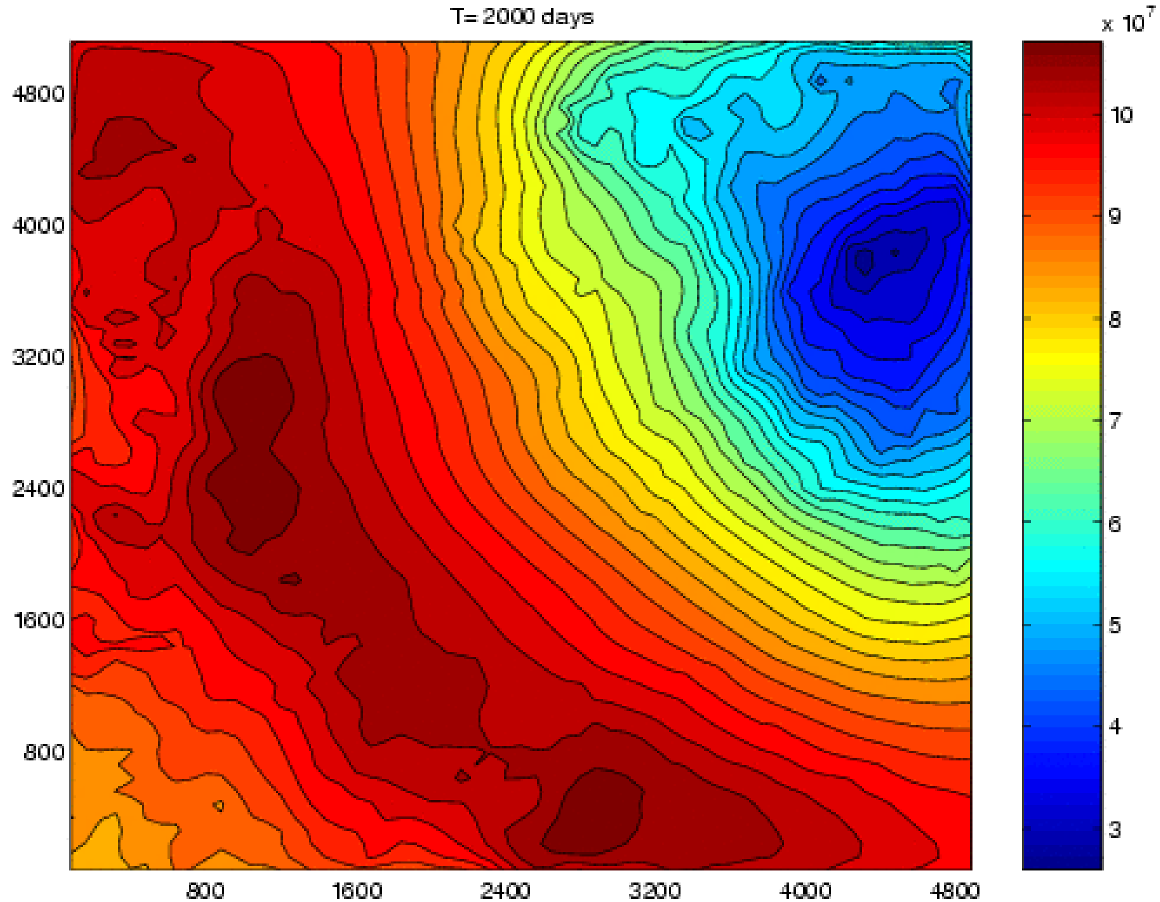
With this compute solution of PDE up to time *T*.

- From solution, extract oil/water flow rates
- Compute economic value by taking into account flow rates, price of oil, cost of water injection, cost of water disposal, interest rates

Final result: Net Present Value (NPV) of a field given the chosen locations of wells!

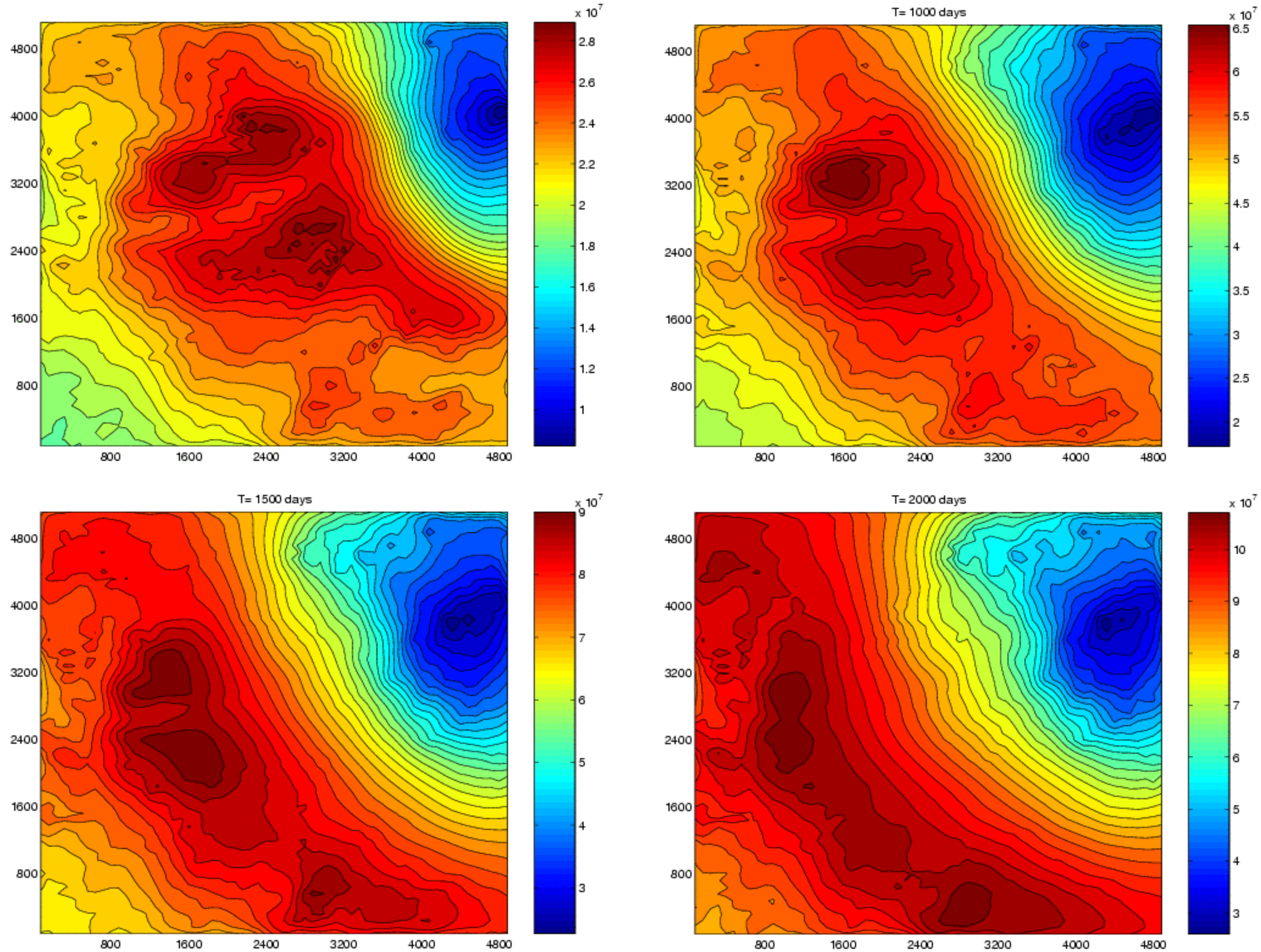
Example of Solution Surface

Determination of optimal well location by brute force:
For each possible well location (61x64 grid, i.e. 3904 possible locations) compute the production and injection history, and compute integrated revenue up to time *T*. Plot this as a function of well location:



Example of Solution Surface

If we are not sure how long we want to produce the field, we can generate revenue surfaces for different time horizons :



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Problems with PDE Optimization

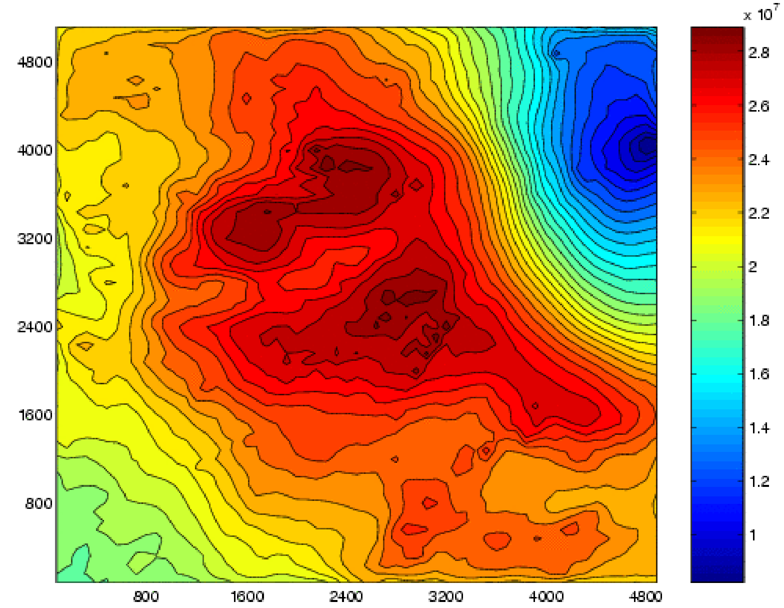
- **Function evaluation is very expensive:** One computation of economic revenue for a given well location takes ~25 minutes
- **We need many evaluations:** To completely characterize the solution surface, we may have to do thousands of computations
- **In the stochastic case:** If only stochastic information on the system is available, we need to sample probability space, increasing the number of computations by a factor of 50-100.

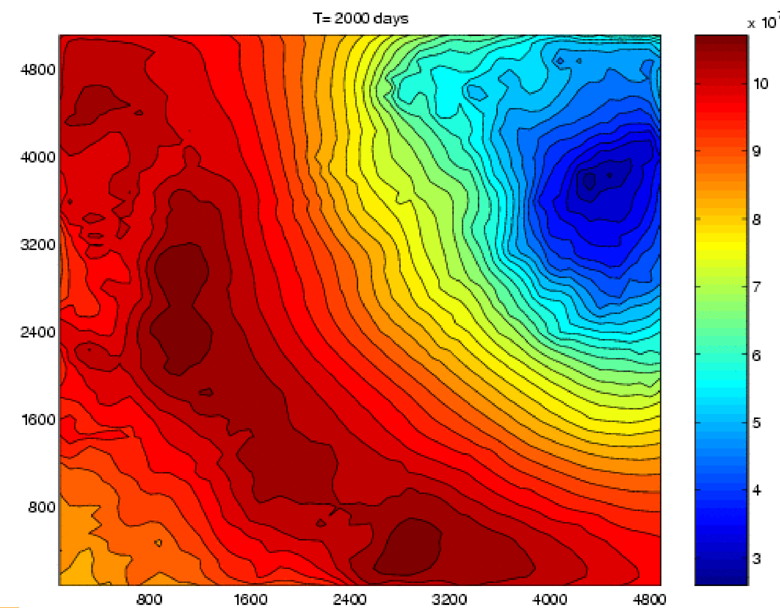
Some solutions:

- Use a good optimizer
- Use clusters of computers
- Use Grid technology

Optimizers

- **Efficient:** Optimizers have to use as few function evaluations as possible
- **Robust:** Should still work in the case of local maxima (at least on average)
- **Accurate:** Switch to local methods close to solution
- **Extendible:** Usable also for problems with uncertainty in the data
- **Parallelizable:** Do as many operations in parallel as possible
- **Factorizable:** Small interface between optimizer and function evaluator



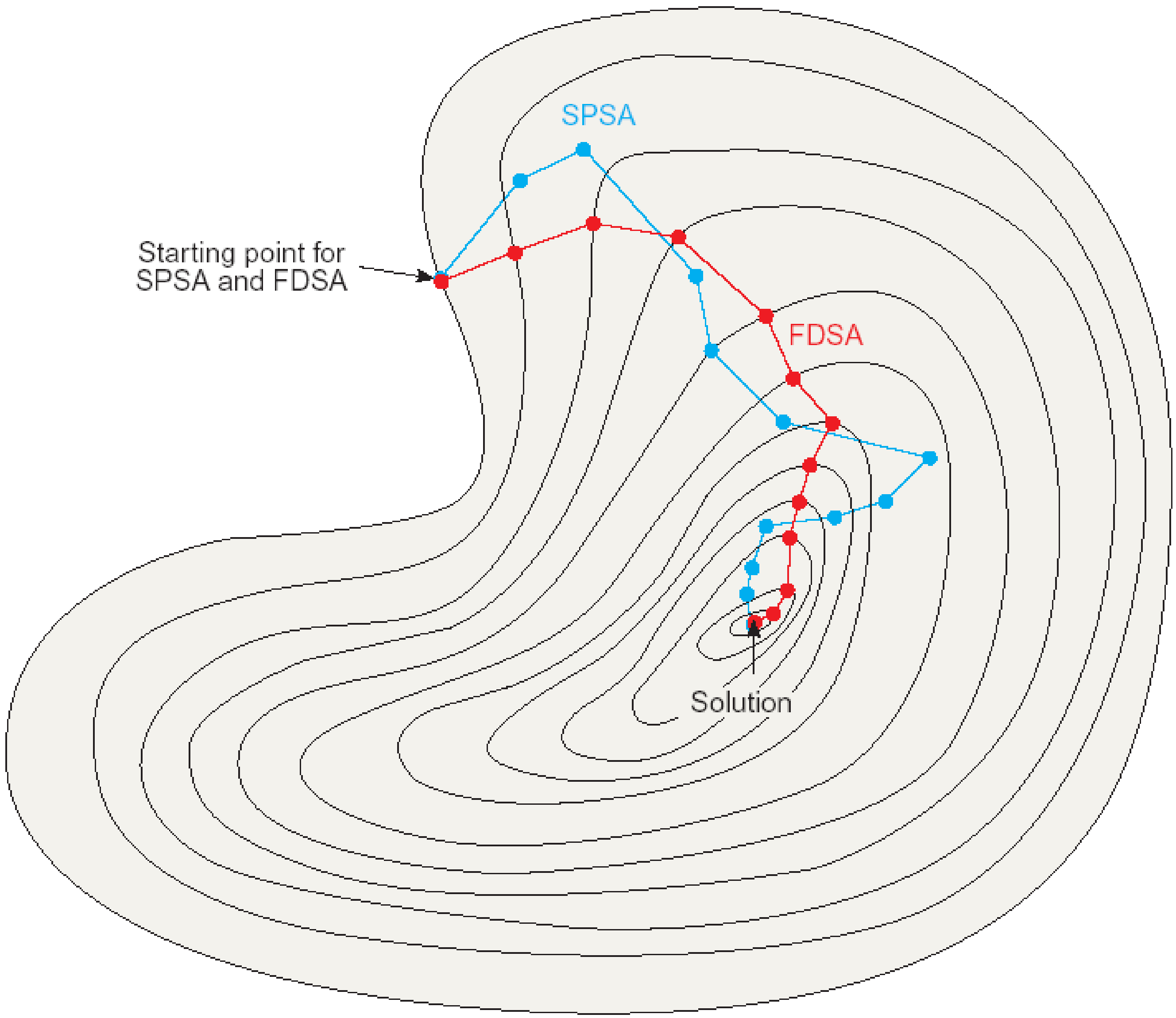


Simultaneous Perturbation Stochastic Approximation (SPSA)

Idea:

- Given present iterate x_k, take a random direction p_k
- If "it goes down in this direction", then make a step in this direction; otherwise go into the other direction
- We need only two function evaluations per step, in comparison to 2N for a finite difference approximation of the gradient
- Every step is an improvement. The direction we take has an angle of less than 90 degrees with the steepest descent direction. Since we choose random directions, they fluctuate around the negative gradient. Experience and theory shows that method is faster than FD approximation of steepest descent method

Simultaneous Perturbation Stochastic Approximation (SPSA)



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(Spall, 2003)

Very Fast Simulated Annealing (VFSA)

Idea:

- Given present iterate x_k, pick a random new point p_k from a random distribution centered at x_k with a diameter that decreases with time
- If the new point is better than the old one, then take it
- If the new point is worse than the old one, then
  - take it with a probability that decreases with the amount by which it is worse, and that decreases with time
  - otherwise reject it and start again at the old point

Compare SPSA and VFSA with a standard Finite Difference Gradient algorithm, the Nelder-Mead simplex algorithm, and a Genetic algorithm.

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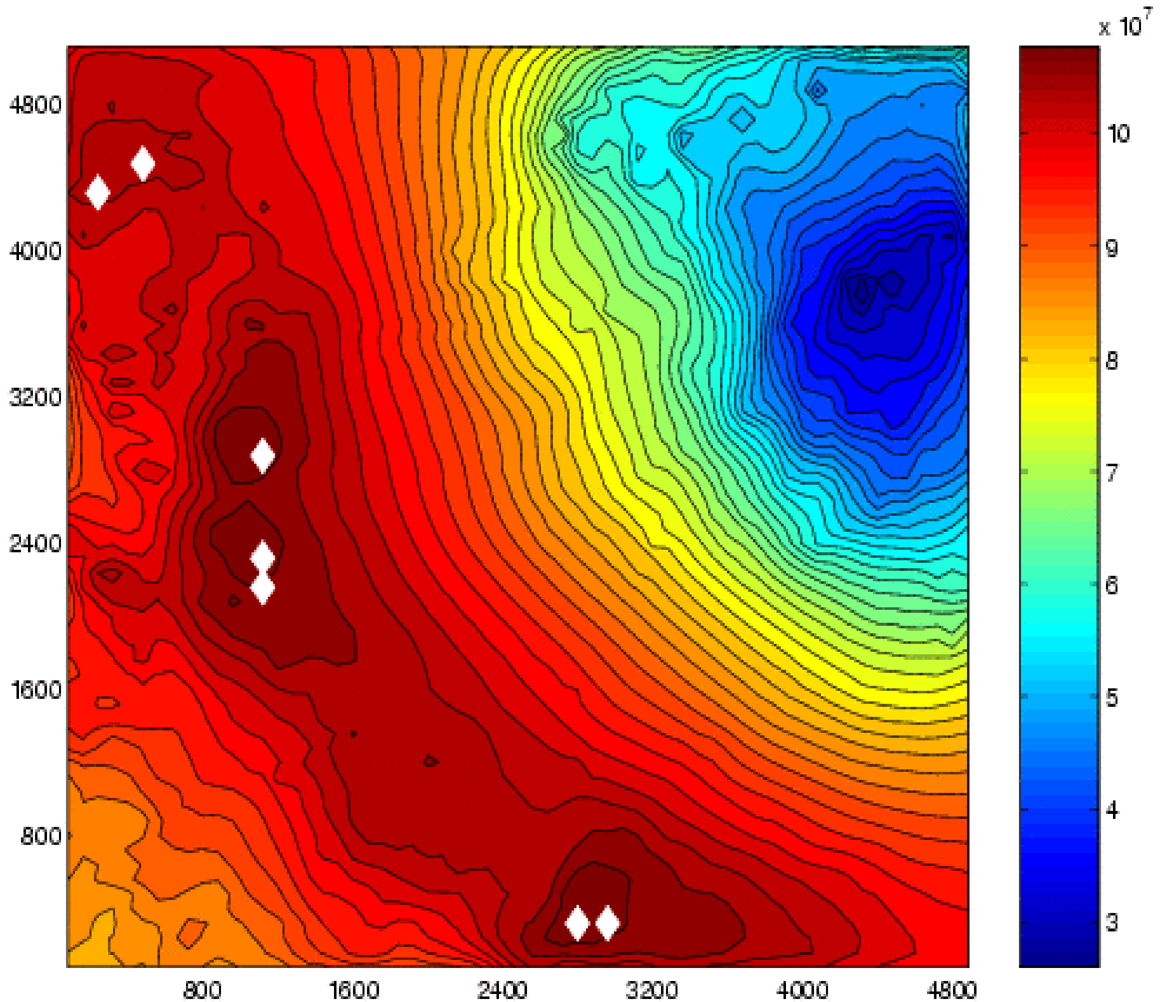
Characterization of algorithms

Since we don't know a priori where a good initial position is, test with several of them and observe where the final positions will be:



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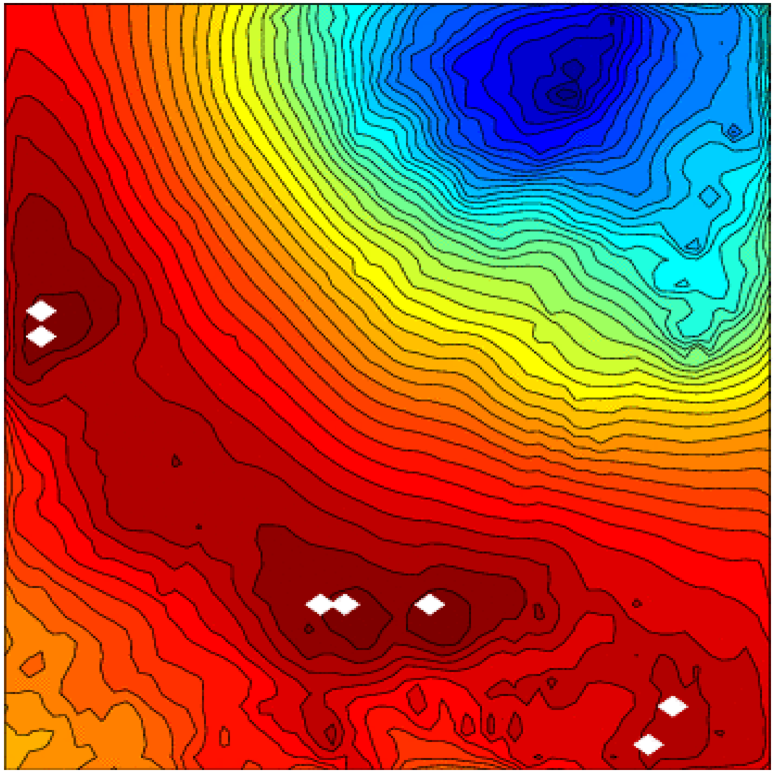
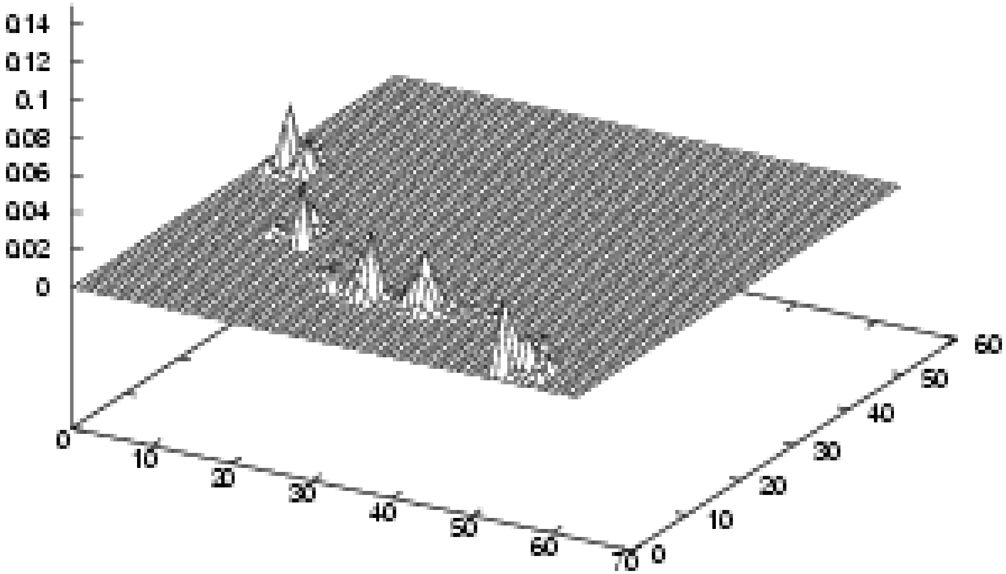
Characterization of algorithms

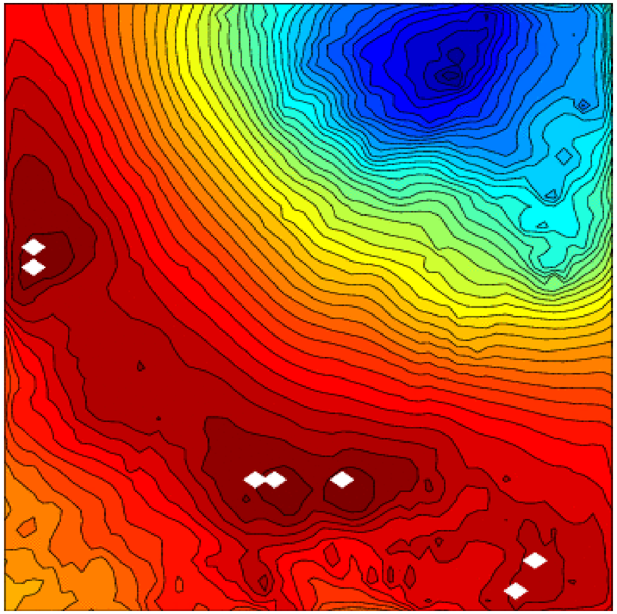
Alternatively, make an experiment where we start from a great number of randomly chosen initial positions (too expensive in reality) and observe the frequency with which we end up at a given final position:

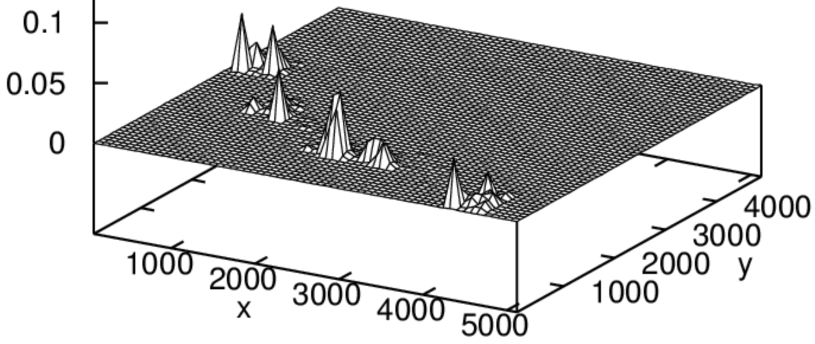




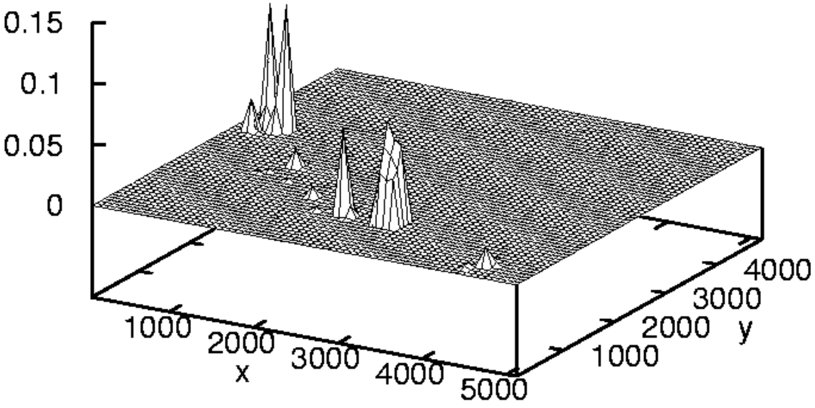
Results single well case, T=2000 days

Revenue surface and probability that SPSA or VFSA stop at a certain well location





SPSA



VFSA

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Results single well case, T=2000 days

- VFSA finds the global maximum most often among all algorithms
- On average,
  - SPSA achieves 98% of the globally best value in 38 function evaluations
  - VFSA achieves 98.6% of the best value in 76 function evaluations
  - the Finite Difference Gradient algorithm achieves 96.7% in 57 function evaluations
- Genetic algorithm and Nelder-Mead are significantly worse

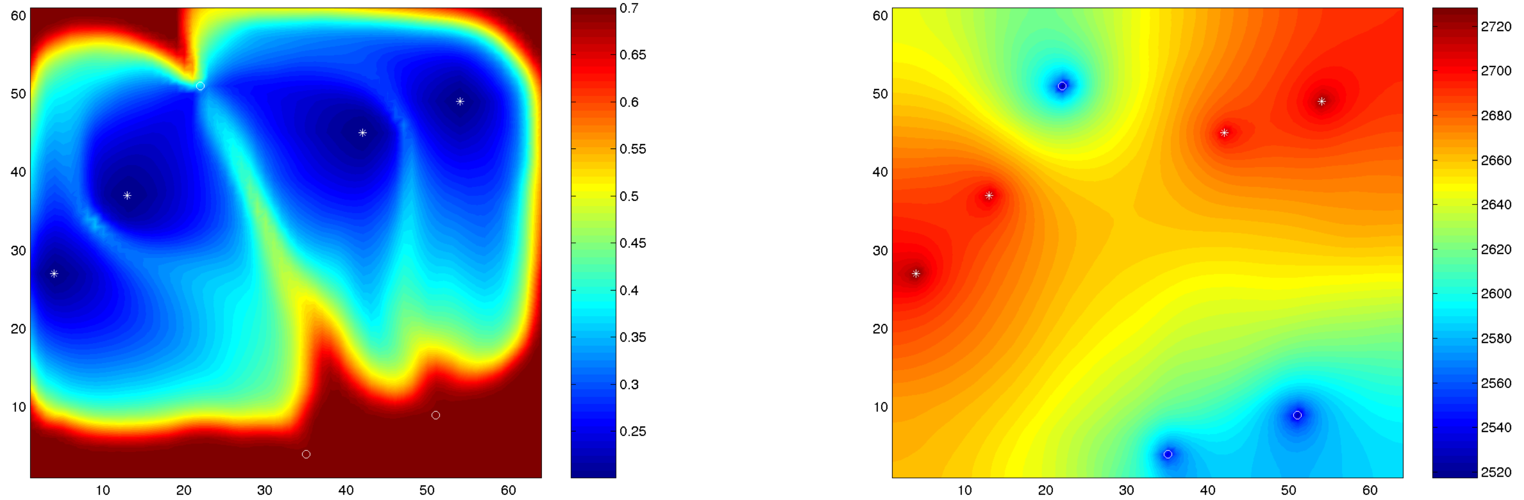
Conclusion of single-well case:

- SPSA most efficient
- VFSA most reliable

Optimizing for multiple well locations

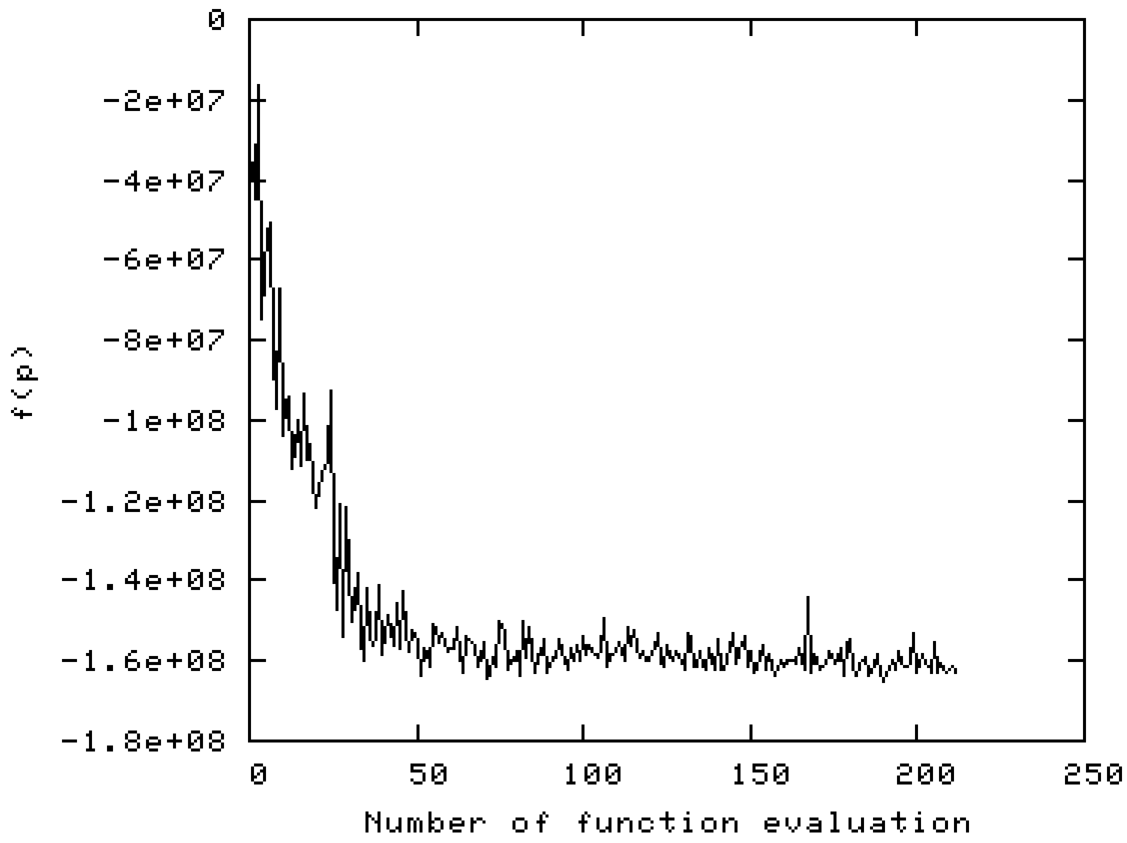
Only moving one well is not interesting. For realistic optimizations we need to find well locations for several wells at once.

As a more interesting testcase, we consider finding the best locations for seven wells in an oil reservoir at once (14-dimensional optimization). Use again SPSA, VFSA, and FDG to compare their efficiency in finding good locations.



Results for SPSA

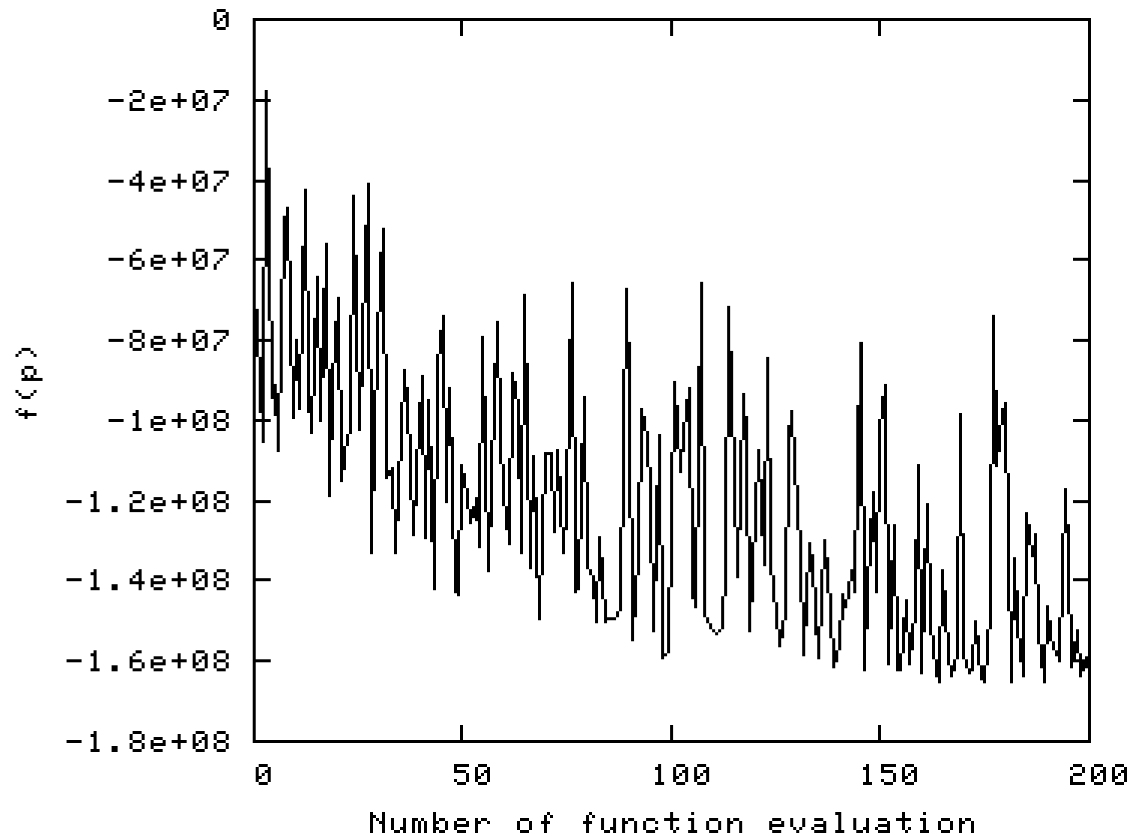
Reduction of objective function by SPSA:



- Best found value: 1.65044 (arbitrary units)
- Finds very good points in as few as 30-50 function evaluations

Results for VFSA

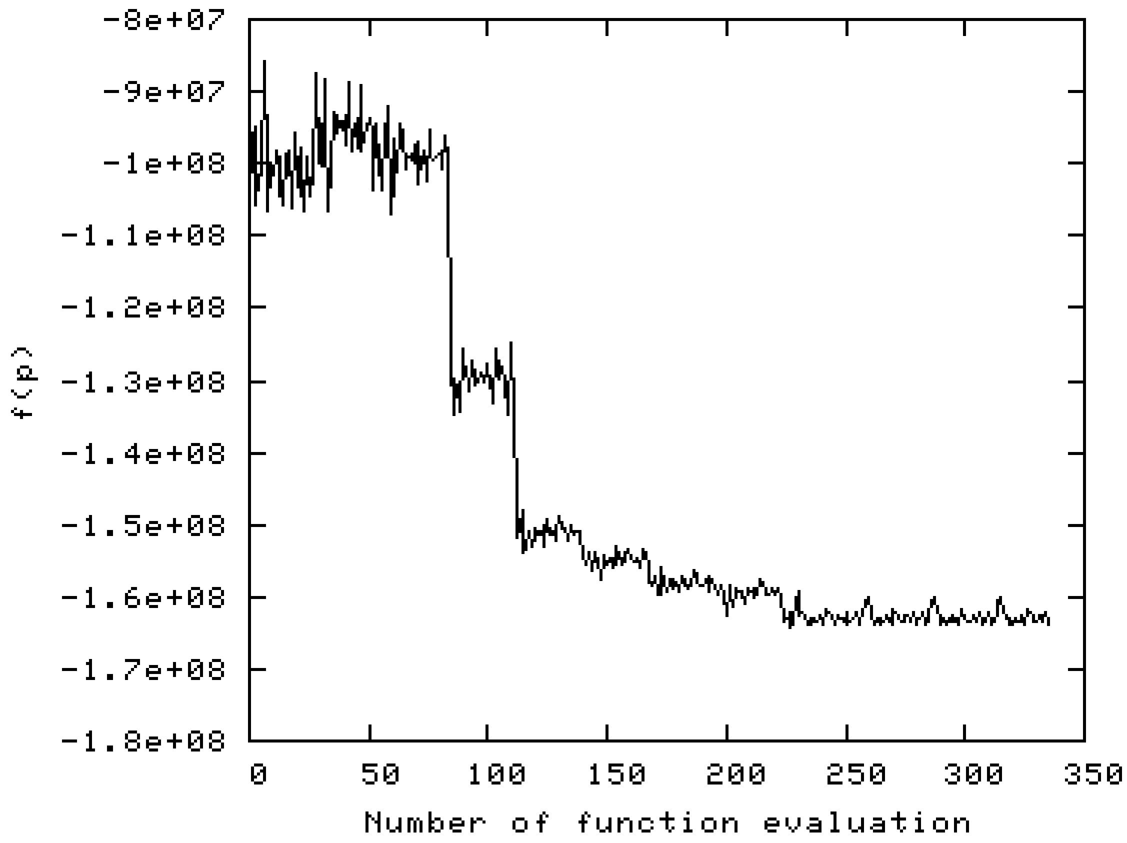
Reduction of objective function by VFSA:



- Best found value: 1.65356 (arbitrary units)
- Finds very good points in 100-150 function evaluations
- Finds good solutions frequently when started from different points

Results for FDG

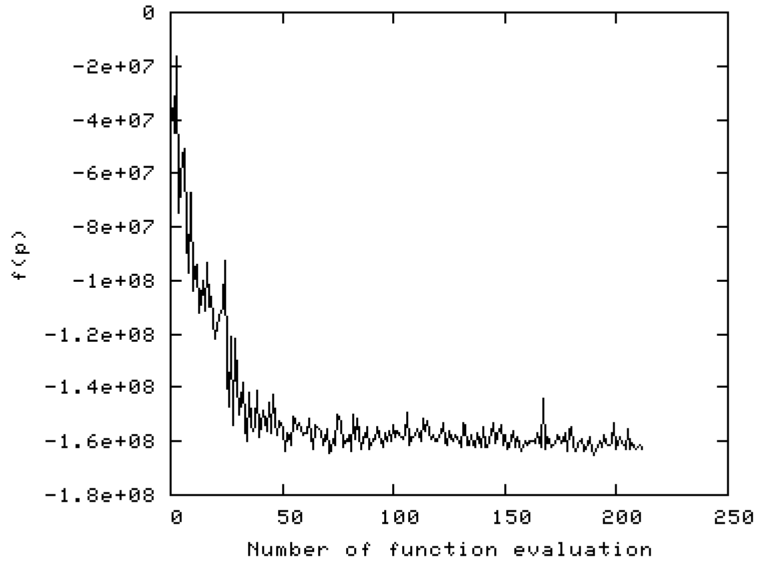
Reduction of objective function by FDG:

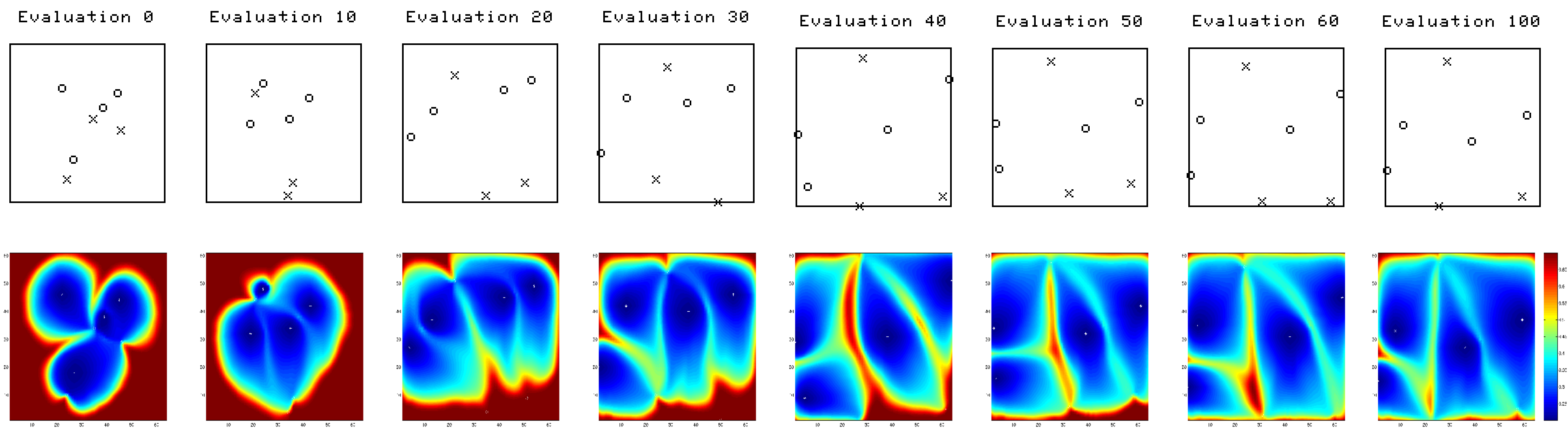


- Best found value: 1.6406 (arbitrary units)
- Finds very good points only in more than 200 function evaluations

Example of optimization process

Reduction in objective function, well locations in different iterations, and oil/water concentrations





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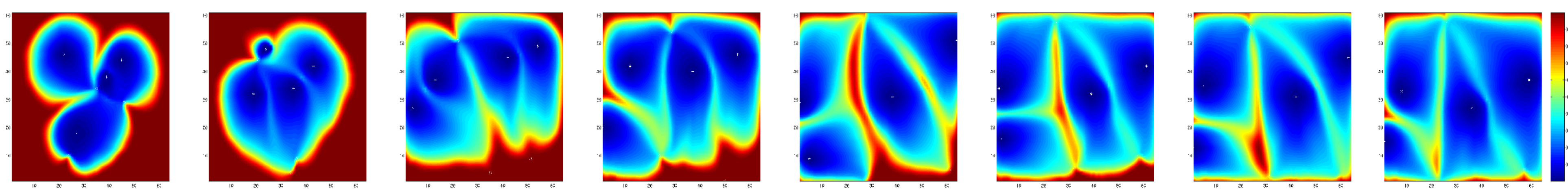
Conclusions and Outlook

- SPSA can find very good solutions even for high-dimensional problems in as few as 50 simulation runs
- VFSA on average finds even better solutions, though at the expense of more function evaluations
- Finite Difference Gradient and other algorithms are worse
- Results can be improved by having parallel runs from different starting positions

- Grid technology significantly reduces computing time
- Future work on more and different control parameters and incorporating geologic and economic uncertainty



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